Continuous Graph Cuts for Prior-Based Object Segmentation *

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Abstract

In this paper we propose a novel prior-based variational object segmentation method in a global minimization framework which unifies image segmentation and image denoising. The idea of the proposed method is to convexify the energy functional of the Chan-Vese method in order to find a global minimizer, so called continuous graph cuts. The method is extended by adding an additional shape constraint into the convex energy functional in order to segment an object using prior information. We show that the energy functional including a shape prior term can be relaxed from optimization over characteristic functions to optimization over arbitrary functions followed by a thresholding at an arbitrarily chosen level between 0 and 1. Experimental results demonstrate the performance and robustness of the method to segment objects in real images.

1. Introduction

Object segmentation is one of the most important and challenging processes in computer vision which aims at extracting objects of interest from a given image. The segmentation results are then used as input for many applications such as recognition, tracking, and classification. The object of interest may exhibit variability in pose and shape which makes segmentation difficult and still a major topic of research.

Many approaches have been proposed to solve the object segmentation problem. In particular, variational methods for image segmentation have had great success such as snakes [8], geodesic active contours [3], geodesic active region [12] and the Chan-Vese method [4]. Yet, the main drawback of those methods is the existence of local minima due to non-convexity of the energy functionals. Minimizing those functionals by gradient descent methods makes the initialization critical. A number of methods have been proposed to find global minima such as [1, 5, 2]. Their approaches give promising results, but it is unclear how to integrate shape constraints.

Integrating shape priors into active contour methods has been proved to improve the robustness of the segmentation methods in the presence of occlusions, clutter, and noise in the images. Various methods have been proposed to address shape prior integration into segmentation process such as [9, 6, 15, 7, 13, 14] and the references therein.

This paper suggests a novel variational approach to prior-based segmentation by adding a shape prior into the global minimization framework using the Mumford-Shah [11] and the Chan-Vese [4] models. The segmentation process is carried out concurrently with the denoising of the image and the transformation of the shape prior to the object of interest. The idea of the proposed method is to use the relaxation of the non-convex energy functional of the Chan-Vese model to the minimization over all functions in such a way that the minimizer of the extended functional can be transformed into the minimizer for the original model by simple thresholding as done in [5]. This method is often called continuous graph cuts and the relaxed energy functional can then be minimized by gradient descent methods to find a global minimum. The main contribution of this paper is to extend this method to also include a shape prior term while maintaining the relaxation property.

2. Continuous Graph Cuts

Minimizing the variational formulation of the Chan-Vese method [4] by gradient descent methods can get stuck in local minima due to the non-convexity of the energy functional. In order to overcome this, Chan et al. [5] propose to convexify the energy functional of the Chan-Vese method [4]. By introducing an auxiliary variable u, the Chan-Vese method can be reformulated

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as the following non-convex minimization problem

$$\min_{u=\mathbf{1}_{\Sigma(x)}} \Big\{ E_s(u) = \int_\Omega |\nabla u| + \lambda \int_\Omega \Big\{ \big(c_1 - I(x)\big)^2 - \big(c_2 - I(x)\big)^2 \Big\} u \, dx \Big\}, \tag{1}$$

where $\mathbf{1}_{\Sigma(x)}$ denotes a characteristic function of a subset Σ of Ω and $\lambda, c_1, c_2 \in \mathbf{R}$ and $I(x)$ is the given image. In the next step (1) is relaxed to the convex problem

$$\min_{0 \le u \le 1} E_s(u), \tag{2}$$

where now u is an arbitrary function bounded between 0 and 1. If $u(x)$ is a minimizer of (2), then the set $\Gamma(\mu) = \{x \in \Omega, u(x) > \mu\}$ has to be a minimizer of the Mumford-Shah functional [11] for a.e. $\mu \in [0, 1]$, implying that the solution to (1) can be obtained by thresholding u at an arbitrary threshold between 0 and 1, for details see [5]. By having a convex energy functional, we can get a global minimum by using a standard gradient descent method. Notice that (2) is not strictly convex which means that it can have several global minima.

3. Shape Priors for Continuous Graph Cuts

We would now like to introduce an additional shape prior term into (1) and the natural choice is to use a shape prior energy of the form

$$E_p(u) = \int_\Omega (u - \mathbf{1}_{\Omega_p})^2 \, dx, \tag{3}$$

where $\mathbf{1}_{\Omega_p}$ is the characteristic function of the shape prior template. Inspired by the fact that $\nabla E_p(u) = 2(u - \mathbf{1}_{\Omega_p})$, we consider the minimization problem

$$\min_{u=\mathbf{1}_{\Sigma(x)}} \Big\{ E_{sp}(u) = E_s(u) + \gamma \int_\Omega \big(\hat{u} - \mathbf{1}_{\Omega_p}(x)\big) u \, dx \Big\}, \tag{4}$$

where $\gamma \in \mathbf{R}$ and $\hat{u}$ is a 'frozen' u which is updated after finding a solution to (4). We further relax (4) to

$$\min_{0 \le u \le 1} E_{sp}(u). \tag{5}$$

Note that (5) still preserves the convexity of (2) with respect to u. The reason for not using u directly is that we would like to preserve the property that the solution to (4) can be obtained from the solution to (5) by a simple thresholding. Also note that the shape prior term $\big(\hat{u} - \mathbf{1}_{\Omega_p}(x)\big)$ has the property that it is positive on $\{\hat{u} \ge \mu\} \backslash \Omega_p$ pushing u to zero and negative on $\Omega_p \backslash \{\hat{u} \ge \mu\}$ pushing u to one, which is driving the set $\{\hat{u} \ge \mu\}$ towards Ω_p, see Figure 1. We are now ready to prove the main theorem of the paper:

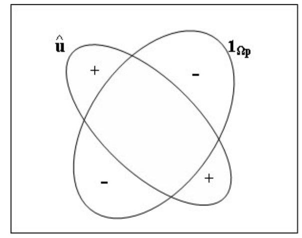

Figure 1. Motivation for using the shape prior as in (4)

Theorem 1. *For any given $c_1, c_2 \in \mathbf{R}$ and $\hat{u} \in \mathbf{R^2}$, a global minimizer of* (4) *is also a global minimizer of* (5).

Proof. We use the coarea formula and the proof in [5] with additional shape prior term

$$\int_\Omega (\hat{u} - \mathbf{1}_{\Omega_p}) \, u(x) \, dx = \int_0^1 \int_{\Omega \cap \{x : u(x) > \mu\}} (\hat{u} - \mathbf{1}_{\Omega_p}) \, dx \, d\mu.$$

Setting $\Gamma(\mu) := \{x : u(x) > \mu\}$, for any $u(x) \in L^2(\Omega)$ such that $0 \le u(x) \le 1$ for all $x \in \Omega$, we have (5) is equal to

$$\min_{u, c_1, c_2} \Big\{ \int_0^1 \Big\{ \mathrm{Per}(\Gamma(\mu); \Omega) + \lambda \int_{\Gamma(\mu)} (c_1 - I(x))^2 \, dx + \lambda \int_{\Omega \backslash \Gamma(\mu)} (c_2 - I(x))^2 \, dx + \gamma \int_{\Gamma(\mu)} (\hat{u} - \mathbf{1}_{\Omega_p}) \, dx \Big\} d\mu - C \Big\},$$

where $\int_0^1 \mathrm{Per}(\Gamma(\mu); \Omega) \, d\mu = \int_\Omega |\nabla u|$ and $C = \int_\Omega (c_2 - I(x))^2 \, dx$ is independent of u. It follows that if $u(x)$ is a minimizer of (5), then it is also a minimizer of (4). □

Corollary 1. *The solution to (4) can be obtained from the solution to (5) by thresholding at an arbitrary level between* 0 *and* 1.

In order to make (5) invariant with respect to similarity transformations, the convex minimization problem of (5) is reformulated by adding transformation parameters, as in [6], with the respect to the shape prior $\mathbf{1}_{\Omega_p}$

$$\min_{0 \le u \le 1, s, \theta, T} \Big\{ E_s + \gamma \int_\Omega \big(\hat{u} - \mathbf{1}_{\Omega_p}(s \, R \, x + T)\big) \, u \Big\}, \tag{6}$$

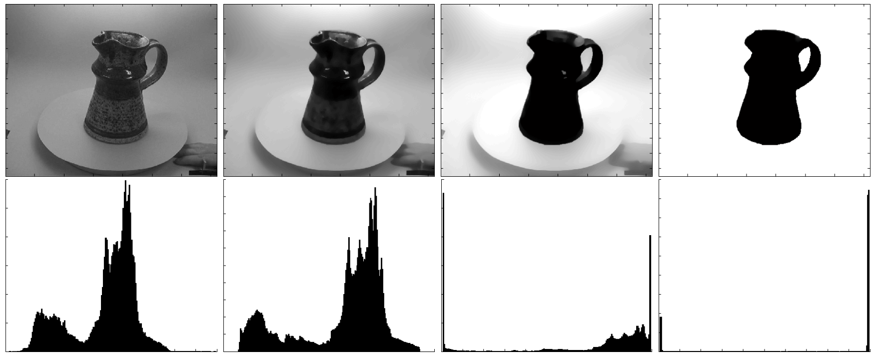

Figure 2. The evolution of $u(x)$ (top row) and the corresponding histogram (bottom row). First column: initial. Middle columns: intermediate results. Right column: final results

for the scaling s, translation T, and rotation matrix $R(\theta)$. Notice that the minimization problem of (6) is no longer convex in all unknowns, but the convexity with respect to u facilitates the computation of the transformation parameters. To minimize (5), the constrained minimization problem is reformulated as the unconstrained minimization problem by adding a penalty term $\nu(u)$

$$\min_u \Big\{ E^e_{sp}(u) = E_{sp}(u) + \alpha \int_\Omega \nu(u) \Big\}, \quad (7)$$

where $\nu(\xi) := \max\{0, 2|\xi - \frac{1}{2}| - 1\}$ and $\alpha > \frac{\lambda}{2}\|(c_1 - I(x))^2 - (c_2 - I(x))^2\|_{L^\infty(\Omega)}$. This procedure is exactly the one used in [5] and it can be proven in the same way that a solution to (7) is also a solution to (5). The function $\nu(\xi)$ is then regularized as $\nu_\epsilon(\xi)$ by a small constant $\epsilon > 0$ to smooth the sharp bend at 0 and 1.

4. Implementation and Results

The proposed energy functional (6) is minimized with respect to u and the transformation parameters s, R, T by gradient descent method. The evolution equations associated with the Euler-Lagrange equations of (6) are the following

$$\begin{aligned}
\frac{\partial u}{\partial t} &= \nabla \cdot \Big(\frac{\nabla u}{|\nabla u|}\Big) - \alpha \nu_\epsilon^{'}(u) \\
&\quad - \lambda \Big(\big(c_1 - I(x)\big)^2 - \big(c_2 - I(x)\big)^2 \Big) \quad (8) \\
&\quad + \gamma \big(\hat{u} - \mathbf{1}_{\Omega_p}(s\,R\,x + T) \big), \\
\frac{\partial s}{\partial t} &= \gamma \int_\Omega \nabla \mathbf{1}_{\Omega_p}(s\,R\,x + T)\, u\, R\, x\, dx, \quad (9) \\
\frac{\partial \theta}{\partial t} &= \gamma \int_\Omega \nabla \mathbf{1}_{\Omega_p}(s\,R\,x + T)\, u\, s\, \frac{dR}{d\theta}\, x\, dx, \quad (10) \\
\frac{\partial T}{\partial t} &= \gamma \int_\Omega \nabla \mathbf{1}_{\Omega_p}(s\,R\,x + T)\, u\, dx, \quad (11)
\end{aligned}$$

where $\nu_\epsilon^{'}(u)$ denotes the derivative of $\nu_\epsilon(u)$. Here the gradient descent of u (8) is coupled with gradient descents of the transformation parameters which update dynamically the transformation parameters to map $\mathbf{1}_{\Omega_p}$ and $\hat{u}$ in the best possible way (see Algorithm 1).

Algorithm 1 Algorithm for minimizing the proposed segmentation functional

INPUT: $I, u, \mathbf{1}_{\Omega_p}, s, \theta, T, \alpha, \lambda, \gamma, \Delta t$
OUTPUT: Optimal u

1. Compute c_1 and c_2 as mean intensities of region inside and outside the contour.
2. Compute the transformation parameters using (9), (10), and (11).
3. Transform the prior.
4. Update u using (8).
5. Repeat until convergence

We implement the proposed method to segment objects in images. As shown in Figure 2, $u(x)$ takes values between 0 and 1 during the evolution and at convergence, it is very close to being binary. The values of $u(x)$ at the end accumulate near 0 and 1, as shown in the histograms of $u(x)$. The regularized exact penalty term $\nu_\epsilon(\xi)$ in (7) prevents them to be 0 and 1. Figure 4 shows the segmentation results of a bird [10] and a cup. The given images are used as the initial of $u(x)$ and the contours are represented by $u(x) = 0.5$. As we can see, at the convergence state, global minima are found and the segmentation results can then be improved to segment objects of interest by adding shape priors, which are segmented manually and are then transformed to different pose, despite the presence of occlusions.

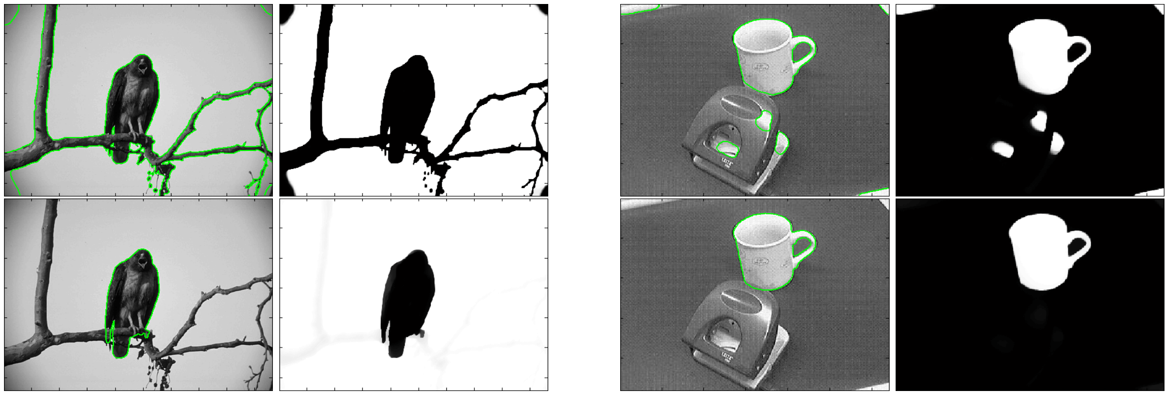

Figure 3. Segmentation of a bird and a cup using continuous graph cuts and the corresponding $u(x)$. Top row: without a shape prior. Bottom row: with a shape prior.

5. Conclusions

We have proposed a novel variational region-based active contour method for prior-based object segmentation in a global minimization framework. The method is based on convexifying the energy functional of Chan-Vese method and adding a shape prior term as a constraint to segment an object whose global shape is given. The energy functional can be relaxed from optimization over characteristic functions to over arbitrary functions followed by a thresholding at an arbitrarily chosen level between 0 and 1. Experimental results demonstrate desirable performance of the method to segment objects of interest in the images.

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