A Novel Stability Quantification of Detected Interest Points in Scale-Space

Alaa E. Abdel-Hakim
Electrical Engineering Department
Assiut University, Assiut, Egypt
alaa.hakim@ieee.org

Aly A. Farag
CVIP Laboratory, University of Louisville
Louisville, KY USA
farag@cvip.uofl.edu

Abstract

In local invariant features, the detected interest points using scale-space representations are considered the most robust to many variations in the imaging conditions. The existing approaches extract interest points at scale-space differential singularities. In these approaches, all detected interest points are considered equally robust without taking in consideration the variable sensitivities of these interest points with respect to noise effects. In this paper, we analyze the robustness of detected interest points at scale-space singularities against noise effects. Also, we propose a novel quantitative stability measure of these interest points. The evaluation results show the effectiveness of the proposed stability measure in estimating the robustness of the detected interest points to noise effects.

1 Introduction

Object description using local invariant features is one of the most widely used methodologies in many matching-based applications. The first step in extracting local invariant features is detecting a set of points at which to build feature descriptors. Several interest points detectors have been developed in the literature, e.g. Canny edge detector [1], Harris corner detector [3], Laplacian pyramids [5], Harris-Laplacian detector [7].

The robustness of the detected interest points with respect to different geometric distortions is one of the most crucial factors which must be considered in feature extraction processes. Scale-space representation provides a good stability against different sources of variations.

Therefore, some well-known approaches, e.g. [6], use scale-space to detect the most stable interest points. These kinds of approaches treat all detected interest points in scale-space similarly. In other words, the detected interest points are not distinguished, although they do not have the same degree of robustness, especially with respect to noise effects. This lack of stability quantification of the detected interest points may negatively affect the performance of the matching process. To our knowledge, none of the existing approaches in the literature tried to address this problem.

In this paper, we analyze the effect of the noise on the stability of the detected interest points. Then, we propose a novel stability measure for numerical estimation of the robustness of the detected interest points in scale-space.

2 Interest Point Detection in Scale-Space

Scale-space theory offers the main tools for selecting the most robust feature locations, or the interest points, against scale variations [5,9]. Given a signal $f : \mathbb{R}^N \rightarrow \mathbb{R}$, the scale-space representation $L : \mathbb{R}^N \times \mathbb{R}_+ \rightarrow \mathbb{R}$ is defined as:

$$L(\mathbf{x}, t) = g(\mathbf{x}, t) * f(\mathbf{x}) \quad (1)$$

where $L(\mathbf{x}, 0) = f(\mathbf{x}) \forall \mathbf{x} \in \mathbb{R}^N$ and $g(\mathbf{x}, t)$ is the Gaussian scale-space kernel [5].

The absolute invariance with respect to uniform rescalings of the spatial coordinates is achieved by detecting the features at the singularities of differential expressions of scale-space [5]. Therefore, scale-space-based interest point detectors detect interest points at differential singularities of scale-space [6].

3 Interest Points Stability Criteria

In this section, we discuss two main criteria which are used to judge the stability of detected interest points: uniqueness of the interest points with respect to changes in the scale parameter and the repeatability of the interest points under different deformations [5].

3.1 Uniqueness

For scale invariance, feature descriptors must consider the scale parameter of the input scale-space signal $L(\mathbf{x}, t)$. So, a vital property of the detected interest points is their uniqueness with the variation of the scale parameter (t). Lindeberg [5] has presented scale-space *'critical non-degenerate points'* as the scale-space points which satisfy this uniqueness constraint.

Definition 1 (*Critical Points*)**.** A point $\mathbf{x}_0 \in \mathbb{R}^N$ is said to be a critical point of a mapping $f : \mathbb{R}^N \rightarrow \mathbb{R}$, if the gradient of $f(\mathbf{x})$ at $\mathbf{x} = \mathbf{x}_0$ equals zero.

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Definition 2 (*Critical Non-degenerate Point*)**.** A critical point $\mathbf{x}_0 \in \mathbb{R}^N$ of a mapping $f : \mathbb{R}^N \rightarrow \mathbb{R}$ is said to be a critical non-degenerate point, if the Hessian matrix of $f(\mathbf{x})$ at $\mathbf{x} = \mathbf{x}_0$is non-singular.

Scale-space is a causal representation, i.e. no new peaks are created when the scale parameter is increased [4]. So, the following property of critical non-degenerate points is obviously verified [5].

Property 1 (*Uniqueness of Critical Non-degenerate Points Over Scales*)**.** *Let $\mathbf{x}_0 \in \mathbb{R}^{\mathbb{N}}$ be a critical non-degenerate point of a scale space representation $L(\mathbf{x};t)$ at a scale level $t = t_0 \in \mathbb{R}_+$. Then, $\forall I_{t_0} \subset \mathbb{R}_+$ with $t_0 \in I_{t_0}$ $\exists$ a unique scale level $t = t_0 \in I_{t_0}$ at which $\mathbf{x}_0$ is a critical non-degenerate point of $L(\mathbf{x};t)$.*

3.2 Repeatability

Repeatability indicates how robust the detected interest points are with respect to different geometric and photometric variations. Repeatability is defined as the percentage of the total number of interest points that appear at the same locations of the interest points detected before these variations [8].

Scale-space differential singularities achieve scale-space invariance, i.e. they achieve maximum repeatability [5]. According to Definition (1), scale-space critical non-degenerate points are differential singularities of scale-space representations. Hence, critical non-degenerate points are good candidates for interest points that achieve maximum repeatability.

4 Stability Quantification

In this section, we give a detailed mathematical analysis of the noise effect on the detected interest points. Then, we explain our proposed stability measure to quantify the robustness of the detected interest points.

4.1 Noise effect

In presence of noise, it is very important to have an accurate estimation of the robustness of the detected interest points. This importance comes from the fact that many false positives and/or false negatives may exist at "weak" interest points. Hence, the repeatability of the detected interest points decreases.

The following proposition illustrates the importance of such quantitative measure.

Proposition 1 (*Noise and Scale-Space Interest Points*)**.** *Existence of noise in input signals may result in removing existing critical non-degenerate points or creating new critical non-degenerate points in the scale-space representation of these signals.*

Proof 1. *Let $L(\mathbf{x};t) \in \mathbb{R}^N \times \mathbb{R}_+$ be a scale-space representation of an input signal $f(\mathbf{x}) : \mathbb{R}^N \rightarrow \mathbb{R}; \mathbf{x} \in \mathbb{R}^N$.*
Part I: Removal of existing critical non-degenerate points due to noise

Let $(\mathbf{x}_0;t_0) \in \mathbb{R}^N \times \mathbb{R}_+$ be a critical non-degenerate point of $L(\mathbf{x};t)$. Assume that a noise signal $N(\mathbf{x})$ has been added to the original signal $f(\mathbf{x})$, then:

$$L_N(\mathbf{x};t) = g(\mathbf{x};t) * (f(\mathbf{x}) + N(\mathbf{x})), \tag{2}$$

$$L_N(\mathbf{x};t) = g(\mathbf{x};t) * f(\mathbf{x}) + g(\mathbf{x};t) * N(\mathbf{x})). \tag{3}$$

Let $K(\mathbf{x};t) = g(\mathbf{x};t) * N(\mathbf{x})$, then:

$$L_N(\mathbf{x};t) = g(\mathbf{x};t) * f(\mathbf{x}) + K(t). \tag{4}$$

Without loss of generality, assume that $L(\mathbf{x}_0;t_0)$ is a local minimum in a neighborhood $\mathcal{N}(\mathbf{x}_0;t_0)$. For $K(\mathbf{x};t) \geq L(\mathbf{x}_0;t_0) - min[L(\mathbf{x};t) + K(\mathbf{x};t)\forall(\mathbf{x};t) \in \mathcal{N}(\mathbf{x}_0;t_0), (\mathbf{x};t) \neq (\mathbf{x}_0;t_0)]\exists(\mathbf{x}_1;t_1)$ such that $L_N(\mathbf{x}_0;t_0) \geq L_(\mathbf{x}_1;t_1) + K(\mathbf{x};t_1)$. Hence, $(\mathbf{x}_0;t_0)$ is no longer a critical non-degenerate point of $L_N(\mathbf{x}_0;t_0)$, i.e. the interest point $(\mathbf{x}_0;t_0)$ may be missed due to the presence of noise.
Part II: Creation of new critical non-degenerate points due to noise

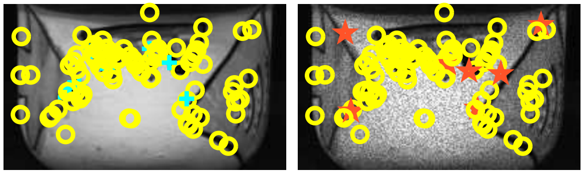
Let $(\mathbf{x}_0;t_0) \in \mathbb{R}^N \times \mathbb{R}_+$ be a non critical point of $L(\mathbf{x};t)$. Therefore, for the local neighborhood $\mathcal{N}(\mathbf{x}_0;t_0)$, and without loss of generality, $\exists(\mathbf{x}_1;t_1)$ such that $L(\mathbf{x}_0;t_0) \geq L(\mathbf{x}_1;t_1)$. Assume that a noise $N(\mathbf{x})$ has been added to the original signal $f(\mathbf{x}_0)$ such that $K(\mathbf{x}_0;t_0)$ satisfies the condition: $K(\mathbf{x}_0;t_0) \geq max[L(\mathbf{x};t); (\mathbf{x};t) \in \mathcal{N}(\mathbf{x}_0;t_0)] - L(\mathbf{x}_0;t_0)$.

Then, after substituting for $K(\mathbf{x}_0;t_0)$ in Eq. (4), $(\mathbf{x}_0;t_0)$ becomes a local maximum of $L_N(\mathbf{x};t)$, i.e. a new critical non-degenerate point has been created in the scale-space representation due to the added noise. A similar argument can be followed for these two parts using differential operators of scale-space.

Figure 1 illustrates the noise effect on scale-space detected interest points. In this example, noise has been added to an original image to produce a noisy image. Then, interest points were detected in both the original and the noisy images at critical non-degenerate points of their scale-space representations. The repeated interested points in both images (true positives) are marked by circles. In compliance with our hypothesis, the existence of noise leads to missing some detected points in the original image (false negatives). These missed points are marked by crosses in Figure 1(a). Also, the noise causes new interest points to be created in the noisy image. These newly created interest points (false positives) are marked by stars in Figure 1(b).

4.2 A novel quantitative stability measure

In this subsection, we propose a novel quantitative measure for stability of the detected interest points at critical non-degenerate points of scale-space representations.



(a) Noise-free (b) Noisy

Figure 1. The noise effect.

Lemma 1 (*Interest Points: Stability Measure*)**.** *Let* $(\mathbf{x}_0; t_0) \in \mathbb{R}^N \times \mathbb{R}_+$ *be a critical non-degenerate point of a scale-space representation* $L(\mathbf{x}; t)$ [1]*. Let* $\mathcal{N}(\mathbf{x}_0; t_0)$ *be a* $d^N \times d$ *neighborhood of* $(\mathbf{x}_0, t_0)$ [2]*. Define a vector* $V = [L(\mathbf{x}; t)]_{(\mathbf{x};t) \in \mathcal{N}(\mathbf{x}_0;t_0)} \in \mathbb{R}^{d^N+1}$ *such that* $V(n_0) = L(\mathbf{x_0}; \mathbf{t_0}), V(n_0 - i) > V(n_0) < V(n_0 + i); n_0 = \frac{d^{N+1}+1}{2}$, $V(n_0 - i) < V(n_0 - i - 1)$ *and* $V(n_0 + i) < V(n_0 - i - 1); i = \{1, ..., n_0 - 1\}$ [3]*. Let* $v(\tau)$ *is a curve fit to the element of* V *such that* $v(\tau*) = V(n_0)$*.*

Then, the value of the curvature of $v(\tau)$ *at* τ^*, $\kappa(\tau^*)$, *gives a quantitative measure for the stability of* $(\mathbf{x}_0; t_0)$ *as a detected interest point in the scale-space representation with respect to noise.*

Proof 2. *The curvature* $\kappa(\tau)$ *is defined by:*

$$\kappa(\tau) = \frac{\frac{\partial^2 v(\tau)}{\partial \tau^2}}{[1 + (\frac{\partial v(\tau)}{\partial \tau})^2]^{1.5}}. \quad (5)$$

Given:

$$\frac{\partial v(\tau^*)}{\partial \tau} = 0, \quad (6)$$

Then:

$$\kappa(\tau^*) = \frac{\partial^2 v(\tau^*)}{\partial \tau^2}. \quad (7)$$

Using finite difference approximation for the second directive:

$$\kappa(\tau^*) = \frac{v(\tau^* + \Delta\tau) + v(\tau^* - \Delta\tau) - 2v(\tau^*)}{(\Delta\tau)^2}. \quad (8)$$

For simplicity, assume that $\Delta\tau = 1$*, i.e.* $v(\tau^* + \Delta\tau) = V(n_0 + 1)$ *and* $v(\tau^* - \Delta\tau) = V(n_0 - 1)$*, then:*

$$\kappa(\tau^*) = V(n_0 + 1) + V(n_0 - 1) - 2v(\tau^*). \quad (9)$$

In presence of noise:

$$\begin{aligned}\kappa'(\tau^*) = \quad & V(n_0 + 1) + K(\mathbf{x_1}; \mathbf{t_1}) + V(n_0 - 1) \\ & + K(\mathbf{x_2}; \mathbf{t_2}) - 2[v(\tau^*) + K(\mathbf{x_0}; \mathbf{t_0})], \quad (10)\end{aligned}$$

where $K(\mathbf{x_1}; \mathbf{t_1})$ *and* $K(\mathbf{x_2}; \mathbf{t_2})$ *are the noise values at the closest neighbors of* $(\mathbf{x_0}; \mathbf{t_0})$ *in the vector* V*.*

$$\begin{aligned}\kappa'(\tau^*) = \quad & V(n_0 + 1) + V(n_0 - 1) - 2v(\tau^*) \\ & + [K(\mathbf{x_1}; \mathbf{t_1}) + K(\mathbf{x_2}; \mathbf{t_2}) - 2K(\mathbf{x_0}; \mathbf{t_0})] \quad (11)\end{aligned}$$

$$\kappa'(\tau^*) = \kappa(\tau^*) - [2K(\mathbf{x_0}; \mathbf{t_0}) - K(\mathbf{x_1}; \mathbf{t_1}) - K(\mathbf{x_2}; \mathbf{t_2})]. \quad (12)$$

The critical point τ^**, which corresponds to the critical non-degenerate point* $(\mathbf{x}_0; t_0)$*, is converted to a non-extremum point when its curvature value* $\kappa'(\tau^*)$ *equals zero, i.e. the interest point at* $(\mathbf{x}_0; t_0)$ *is no longer existing.*

Considering the case in which the terms $\kappa(\tau^*) - [2K(\mathbf{x_0}; \mathbf{t_0})$ *and* $K(\mathbf{x_1}; \mathbf{t_1}) - K(\mathbf{x_2}; \mathbf{t_2})]$ *have different signs, as the curvature* $\kappa(\tau^*)$ *increases, the noise level against which the interest point is robust increases and vise versa. Therefore, the value of the curvature* $\kappa(\tau^*)$ *indicates the possibility of eliminating an interest point due to the existence of noise at this interest point. By following a similar argument, it can be shown that curvature indicates the possibility of creating new interest points due to existence of noise.*

In conclusion, the value of the curvature $\kappa(\tau^*)$ *can be considered as a quantitative measure for the stability of detected interest points in scale-space against noise.*

[1] WLOG, consider $L(\mathbf{x_0}; t_0)$ to be a local minimum.

[2] Typically, d equals 3.

[3] In case of local maxima, all the identity signs are reversed.

5 Evaluation

5.1 Experiments design

To evaluate the performance of the proposed stability measure, we used 100 images under different imaging conditions from the Amsterdam Library of Objects Images (ALOI) data set [2]. We calculated the scale-space representations for each of these images. Then, we extracted the critical non-degenerate points of the scale-space representations as the detected interest points. For every interest point, the stability measure is calculated as described in the previous section. A total of about 11,500 interest points with their stability measures were collected in a reference database. Each of these interest points has a label telling the image from which this interest point was extracted.

Two types of noise were applied to every test image: (a) uniform with five different noise levels, 2%, 5%, 10%, 15%, and 20% of the image range (b) Gaussian with five different σ's, 2%, 5%, 10%, 15%, and 20% of the image range were applied to generate about 1,000 noisy images. For every noise signal, interest points and stability measures are collected in a database similar to the reference database. The interest points in each of the built databases are sorted according to their stability measures.

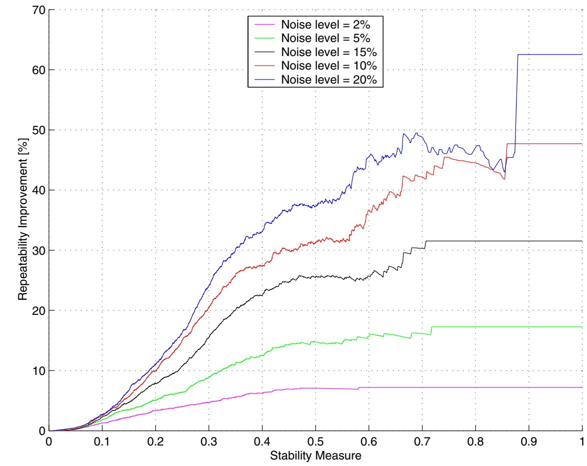
Three quantities are measured for the interest points whose stability measures above certain thresholds: (a) the interest points repeatability of the noisy database with respect to the reference database, (b) the percentage of the missed interest points in the reference database (false negatives) due to the existence of noise, and (c) the percentage of the newly created interest points in the noisy database (false positives).

5.2 Evaluation results

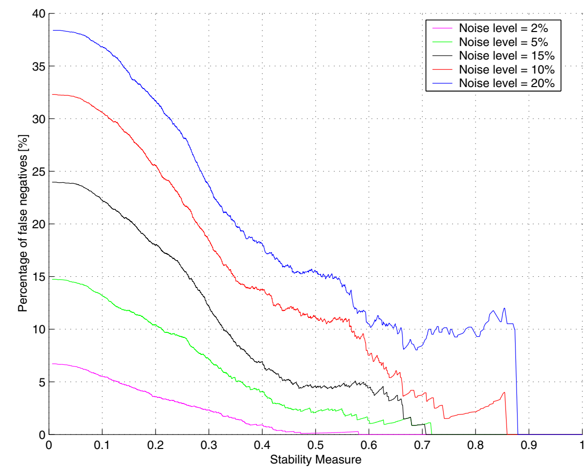
For evaluation purposes, we study the improvement in the repeatability and the decrease of the false positives and false negatives when rejecting weak interest points, i.e. interest points whose stability measure under certain threshold values. Due to space limitations, we show the results of uniform noise only. Figure 2(a) shows the repeatability improvement when using interest points whose stability measure above certain thresholds. It is noted that the repeatability of the detected interest points is improved, when the weak interest points are removed. As the noise in an image increases, the repeatability gets better for the same stability value. This is because the number of the repeated interest points, true positives, becomes smaller.

Figure 2(b) shows the percentage of the false negatives of the noisy databases when interest points of the reference database, whose stability values are under certain thresholds, are discarded. Similar to the repeatability results, preserving stronger interest points leads to getting a smaller fraction of the false negatives. Also, as the noise increases, the ratio of the false negatives decreases. Similarly, the rate of detecting false positives is improved. The improvements in repeatability, false negatives, and false positives positively impact the recognition performance of the adopting object recognition system. Due to space limitation, only the improvements in repeatability and false negatives are shown.

The small disturbance toward the right end of some of evaluation curves occurs due to the large decrease in the number of the considered interest points for larger stability thresholds, since more detected interest points are considered weak ones. Therefore, a good operation range is suggested to be for stability values between 0.4 and 0.55.



(a) Rep. improvement



(b) False negatives

Figure 2. The evaluation results.

6 Conclusions

In this paper, we studied the effect of noise on the stability of detected interest points at scale-space differential singularities. This study showed that added noise may result in producing false positives or false negatives. We proposed a novel stability measure to quantify the robustness of the detected interest points at scale-space differential singularities. The evaluation results proved that the proposed stability measure succeeded in giving a quantitative estimate of the robustness of the detected interest points with respect to noise effects.

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