

Probabilistic Matching of Line Segments for Their Homography

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Abstract

Finding correspondences of two images taken from largely different camera configuration is a challenging problem because appearance information such as color, intensity and edge orientation histogram cannot be used. A probabilistic approach to match line segments in the images is proposed for calculating their homography. A membership matrix to represent contribution of every match to the homography is employed. A relaxed version of forward stepwise regression is derived and the test statistic is optimized. An alternating scheme for optimizing the membership and homography is provided. The simulation results on synthetic images validate the proposed method.

1. Introduction

Finding feature correspondences and matching two images are important tasks that pervade computer vision for motion estimation, object recognition, etc. Various image matching methods depending on the aim and the image features are proposed in a number of applications [12]. Estimating transformation parameter requires feature correspondences in the two images and vice versa. Both correspondences and transformation are estimated simultaneously to solve this chicken-and-egg problem.

Finding correspondences is known to be NP-hard problem and several relaxation techniques are proposed to obtain the approximate solutions. Binary correspondence is relaxed into real value between 0 and 1 so that combinatorial optimization problem is converted into continuous nonlinear optimization. Mathematically, feature set is represented as a graph. Each feature is associated with a node of the graph, and the arc represents their relation. Matching two images is then formulated as searching same structured graph in the two images.

The graph matching problem has been approached in many different ways in the computer vision

[1][2][3][4][5][6][7][8][9][15]. Using a probabilistic relaxation framework Christmas et al. have developed a statistical model for pair-wise attribute relations [1]. The matching problem is formulated in the Bayesian framework for contextual label assignment. The formulation leads to an evidence-combining formula. A dual-step Expectation Maximization algorithm was introduced to match geometric structure in 2D point-sets [3][4]. Two-step iterative process involves coupled operations to estimate point correspondences and geometric transformation parameters. Spectral graph theory was applied to characterize the global structural properties of graphs [4][7][8][9][14]. Spectral graph theory uses the distribution of eigenvalues to provide a compact summary of graph-structure. Those work aimed to realize relational matching by inexact means.

Finding correspondences of two images taken from largely different camera configuration is a critical image interpretation tool for high level vision. In the situation, the appearance information such as color, intensity and edge orientation histogram is useless [10]. We propose a probabilistic approach to match line segments in the images for estimating their homography transformation. For relaxing the discrete correspondence, a membership matrix is introduced to represent contribution of every match to the homography transformation. A membership matrix is interpreted as a probability measure. A relaxed version of forward stepwise regression is derived and the test statistic is optimized. An alternating scheme for optimizing the membership and homography is provided. The simulation results on synthetic images validate the proposed method.

The outline of this paper is as follows. In Section 2, probabilistic matching of line segments is described introducing membership matrix and proposed algorithm. We present experimental result of the proposed method in Section 3. Finally, Section 4 presents some conclusions and suggests directions for further investigation.

2. Probabilistic Matching

Given two sets of features their correspondence and homography are estimated simultaneously. Let P and Q be two sets of data and model features containing n_p and n_q features, respectively. A correspondence mapping is a set C of pairs (or assignments) (i, j) , where $i \in P$ and $j \in Q$. The transformation from data to model features is a homography \mathbf{H} since all features are projected from the same ground plane (Figure 1). An associated score or affinity that measures how well feature $i \in P$ matches $j \in Q$ for a given \mathbf{H} . Finding the optimal correspondence to maximize the affinity in a discrete domain is NP-hard.

A membership matrix \mathbf{C} is defined as a relaxed version of correspondence matrix whose elements c_{ij} for (i, j) take real values in $[0, 1]$. The membership of (i, j) reflects the likelihood of the correspondence for a given homography. Each feature in P and Q cannot abuse its resources so that its total membership, the sum of all membership to all points in the other set, does not exceeds a unit:

$$\begin{aligned} p_i &:= \sum_{j=1}^{n_q} c_{ij} \in [0, 1], \\ q_j &:= \sum_{i=1}^{n_p} c_{ij} \in [0, 1]. \end{aligned} \quad (1)$$

where p_i and q_j be the row and column sums of \mathbf{C} , respectively. Homography \mathbf{H} is estimated to minimize the relaxed sum of squared error:

$$SSE(\mathbf{A}, \mathbf{H}) = \sum_{i,j} c_{ij} SSE(i, j; \mathbf{H}), \quad (2)$$

where $SSE(i, j; \bullet)$ is the squared error due to (i, j) .

Selecting significant variables is very important in the regression model. Variable selection method, which performs forward selection method and backward selection method repeatedly, is usually used in regression analysis [3]. Forward stepwise selection method determines variables by the significance test of a reduced model to full model. The change of F statistic due to that of the membership c_{ij} is defined by

$$\frac{\Delta F}{\Delta c_{ij}} = \frac{MSR(\Delta c_{ij} | c_{ij}, \mathbf{H})}{MSE(c_{ij} + \Delta c_{ij}, \mathbf{H})}, \quad (3)$$

where $MSE(\bullet)$ and $MSR(\bullet | \bullet)$ means mean squared error of full and reduced models, respectively. $\Delta \mathbf{C}$ is the set of variables which are added or deleted. Variable selection method determine discrete $\Delta \mathbf{C}$ which maximizes (3). The relaxed version of (3) induced by small $\Delta \mathbf{C}$ is derived:

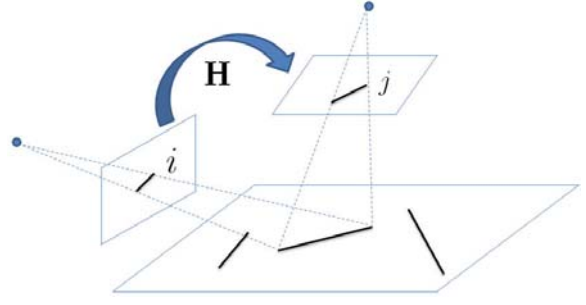


Figure 1. Homography and Correspondence of Line segments

$$\begin{aligned} \frac{\partial F}{\partial c_{ij}} &= \lim_{\Delta c_{ij} \rightarrow 0} \frac{MSR(\Delta c_{ij} | c_{ij}, \mathbf{H})}{MSE(c_{ij} + \Delta c_{ij}, \mathbf{H})} \\ &= (M - \text{dof } \mathbf{H}) \frac{\partial \log SSE(\mathbf{C}, \mathbf{H})}{\partial c_{ij}}. \end{aligned} \quad (4)$$

The scalar function F having derivatives of (4) exists uniquely by the calculus of several variables, but its closed form is not easy to obtain. The following approximate function is adopted:

$$F = (M - \text{dof } \mathbf{H}) \log SSE(\mathbf{C}, \mathbf{H}). \quad (5)$$

To test the feasibility of converged membership matrix, we compare the elements of membership matrix. Therefore, membership \mathbf{C} and homography \mathbf{H} which minimize are considered as optimized membership and homography matrix,

$$(\mathbf{C}^*, \mathbf{H}^*) = \arg \min_{\|\mathbf{H}\|=1} F(\mathbf{C}, \mathbf{H}). \quad (6)$$

An alternating scheme is adopted to obtain homography and membership matrices (Figure 2).

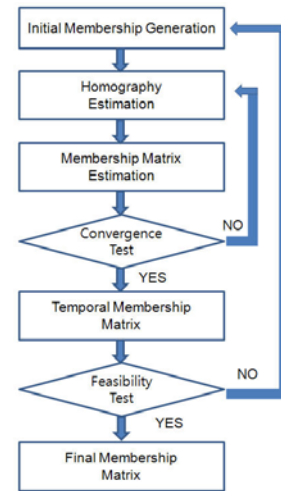


Figure 2. Flowchart of Proposed Algorithm

Homography is estimated from membership matrix and vice versa. Initial membership matrix is estimated from two random pairs of line segments. The optimal solution is chosen from several trials. The membership matrix is distributed uniformly along the row when the correspondence is ill-matched. The feasibility of chosen membership matrix is checked by their standard deviation of each column.

4. Experimental Results

Simulation study is performed for two set of line segments shows correspondences are known (Figure 4). The squared error is defined by the algebraic distance for a given correspondence of line segments [12]. One image has 5 line segments and the other image has 10 line segments. The thick line segments in Figure 4 show the initial and final matches of line segments. As the test statistic in (5) converges, the initial two

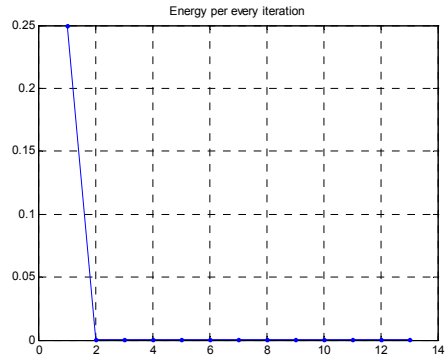


Figure 3. Convergence of F Statistics

matches are propagated to the entire line segment matches (Figure 4 (c)). Convergence of the test statistics is shown in Figure 3.

Figure 5 (a) and (b) show failed and successful

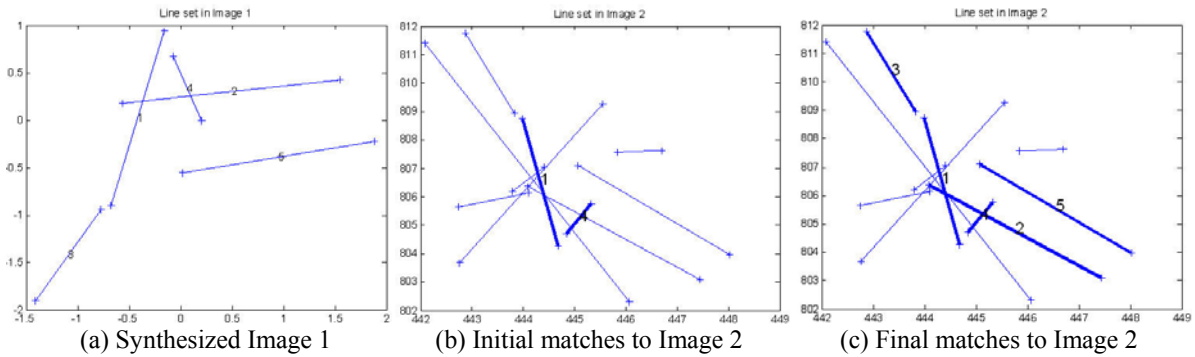
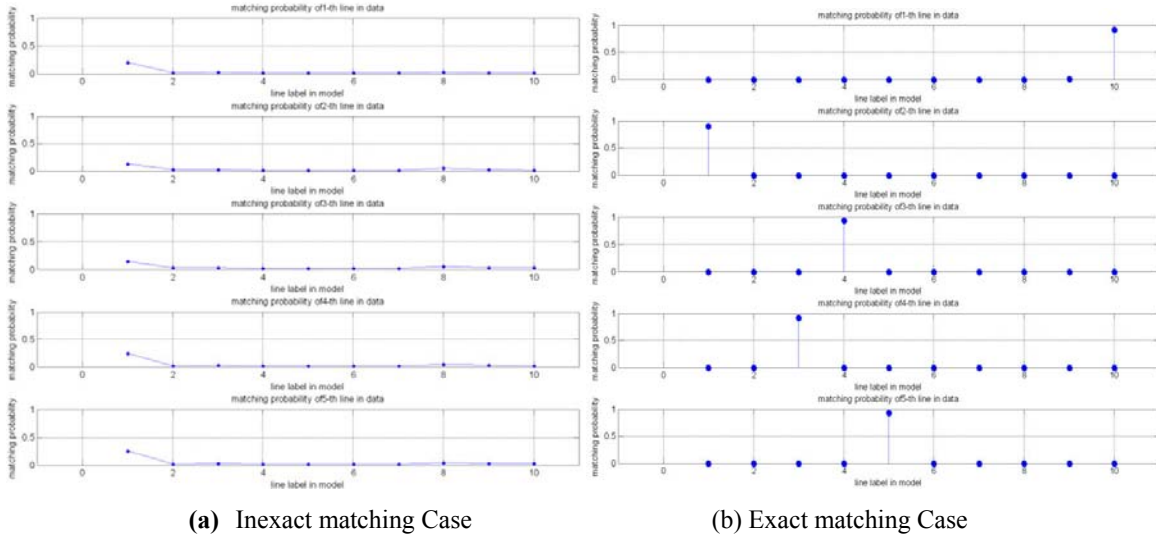


Figure 4. Matching of Line Segments



(a) Inexact matching Case

(b) Exact matching Case

Figure 5. Membership Matrix

cases of matching results. Only one element in a row of membership matrix has almost 1 in the case of optimal matching (Figure 5 (b)). For example, 10th column in the 1st row of the membership matrix has 1. It means 1st line segment in image 1 correspond to 10th line segment in image 2. In the case of bad matching, the membership values are uniformly distributed over all lines with small values Figure 5 (b). Proposed method depends on an initial membership matrix. The feasibility of the final membership matrix is checked by standard deviation of the membership matrix. No threshold is used in proposed algorithm.

5. Conclusion

A new probabilistic approach is proposed to match line segments between two images, taken from largely different camera configuration, for estimating their homography. Discrete characteristic of the line segment correspondence is converted to continuous membership matrix for reducing the computational complexity. It replaces the NP-hard problem with polynomial complexity. An alternating scheme is proposed to optimize homography and membership matrices. A test statistics is proposed as a new error measure.

No threshold is used in the proposed algorithm. This is our main contribution. Regressing process helps to find optimal correspondence without threshold. The test statistics is proposed in this process. This method can be used for satellite and aerial image matching and estimating homography matrix between them. UAVs can estimate their location by comparing captured image and calibrated satellite image through their homography. Developing general matching system which is available to any camera configuration is remained for the future work.

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