Efficient Detection of Projected Concentric Circles using Four Intersection Points on a Secant Line

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Abstract

Concentric circles are often used for calibration. Based on the geometric properties of concentric circles, we proved that only four intersection edge points on one secant line of the two images of the concentric circles (ICC) are sufficient to determine the whole parameters of the ICC, when the ratio of the radii of two concentric circles is given. Experimental results validate the proposed approach.

1. Introduction

Planar calibration patterns are very popular due to their practical convenience. Features on calibration patterns can be grid [18, 10, 11] and circular [5, 1, 8, 3, 13, 14]. This paper employs concentric circles as calibration features because they have richer geometric properties than points and lines. As was shown in [4], a circle is often projected into an ellipse in the image plane, but the projection of the center of the circle is often not the center of the ellipse. For two concentric circles, the centers of their image ellipses are often not coincident. That means the projections of two concentric circles are often not two concentric ellipses. Obviously, the projection of their common center is still one image point in the image plane.

Geometric properties of concentric circles for camera calibration have been studied by many researchers [8, 3]. However, these papers usually used the general ellipse detection methods to detect the images of the concentric circles with general ellipse fitting method [2], not considering the special geometric properties of the concentric circles. Jiang and Quan [6] were first to discuss on the special detection methods for the projected concentric circles. They proposed a constructive method to detect the image of their common center. However, their method cannot deal with the occlusions cases, since it is a recursive procedure and requires the information in the almost whole ellipses. Therefore, this paper aims at the detection of the projected concentric circles with occlusions.

As noted before, the image of the common center of concentric circles is no longer the centers of their image ellipses and cannot be detected using Hough transform based methods [12, 16]. Heikkila [5] utilized a recursive procedure with the rough camera parameters to correct the bias between the imaged centers of circles and centers of ellipses after knowing the latter positions. That means the images of the centers of the circles on the calibration pattern are very useful from the viewpoint of calibration. In the detection methods for the projected concentric circles as emphasized by Jiang and Quan [6], the positions of the imaged common center can be directly located without considering the camera parameters. Though our purpose is to determine ICC, the main idea in this paper arises from observations on detection methods of ellipse based on Hough transform [12, 16, 17].

Standard Hough transform based ellipse detection methods need a 5-dimensional parameter space that consists of the semi-axes, the center point, and the orientation. However, the 5-dimensional parameter space has a huge computational burden. To avoid such computational cost and memory requirements, Yuen et al. [16] used the properties of the ellipse to detect the center in the first step, and other parameters are recovered later. Yoo and Sethi [17] used the pole and polar definition of an ellipse, and its advantage lies in avoiding the propagation of parameter estimation error over the successive stages of the Hough transform.

In our former paper [15] we extended the basic idea on the detection of the ellipse center in [16] to detect the image of the common center of two concentric circles. The basic idea in [15] is illustrated in Figure. 1. A secant line intersects the two ICC at four points. From these four intersection points, four tangent lines can be obtained from some edge detection techniques [7]. The line determine by the two intersection points, P_{12} and P_{34}, from the tangent lines on the same ellipse respectively, must pass through the image of the

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center of the two concentric circles. Note that in this paper we find lines passing through the image of the common center, not find lines passing through the ellipse center as proposed in [16].

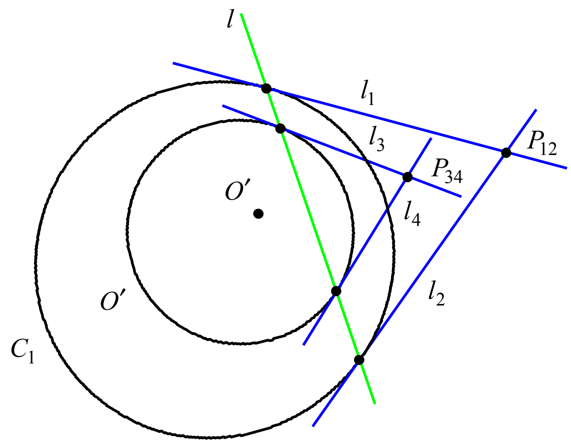


Figure. 1. The geometry of two projected concentric circles. O' is the image of the common center of the concentric circles. C_1 is the image of the outer circle, and C_2 is the image of the inner circle. l is a secant line of the two concentric circles. l intersects the ICC at four points. From these four points, four tangent lines l_1, l_2, l_3, l_4 can be obtained. P_{12} is the intersection point of l_1, l_2. P_{34} is the intersection point of l_3, l_4. We proved that the line $P_{12}P_{34}$ should pass through O' [15].

In this paper, we extend the pole and polar definition of ellipse used by [17] to the ICC. We discovered that only four intersection edge points on one secant line of the two ICC are sufficient to determine the whole parameters of the ICC, when the ratio of the radii of the two concentric circles is given. Note that in [15], since one secant line can only determine one line passing through the imaged center, at least two secant lines are required in this method. The main contribution of this paper is that we use the minimum number, four intersection points on a secant line, not eight points on two secant lines [15], to recover the whole parameters of the two ICC, which makes the proposed method more suitable to deal with serious occlusions.

2. The pole and polar definition of two ICC

The two ICC are shown in Figure. 1. C_1 is the image of the outer circle, and C_2 is the image of the inner circle. Let l be a secant line, which intersects the two ellipses at four points. From these four points, four tangent lines l_1, l_2, l_3, l_4 can be determined as shown in Figure. 1. From [17], we can obtain that the equations of the two ICC are:

$$C_1 = \zeta l^2 + l_1 l_2, \tag{1}$$

and

$$C_2 = \xi l^2 + l_3 l_4, \tag{2}$$

respectively, where ζ and ξ are two unknown constant. Note that ζ and ξ are not arbitrary, since they must make C_1 and C_2 be the images of the two concentric circles. From (1) and (2) we know, to determine C_1 and C_2, we only need to determine ζ and ξ.

In this paper, we assume the equation of l is known, and those of l_1, l_2, l_3, l_4 can be obtained by some edge detection techniques. The ratio of the radii of the two concentric circles is given in advance.

3. Determine the two ICC

In order to solve for ζ and ξ, we rewrite C_1 and C_2 into:

$$C_1 = wl^2 + ul_1 l_2, \tag{3}$$

and

$$\mu C_2 = (1-w)l^2 + vl_3 l_4, \tag{4}$$

where u, v, w, μ are some unknown constant. Obviously, comparing (3) with (1), (4) with (2), we obtain:

$$\zeta = \frac{w}{u}, \tag{5}$$

and

$$\xi = \frac{1-w}{v}. \tag{6}$$

Therefore, to determine the two ICC, we only need to solve for u, v, and w.

3.1. Solve for u, v

From [8], we know, there exists an unknown factor μ, which makes the matrix,

$$C_1 + \mu C_2 = l^2 + ul_1 l_2 + vl_3 l_4 \tag{7}$$

with rank 1. Therefore, all 2×2 submatrices of the matrix $C_1 + \mu C_2$ are singular. Because of the symmetry of $C_1 + \mu C_2$, there are totally 6 equations on u, v from the singularity of these submatrices. Therefore u, v can be solved from these equations using some least squares method.

3.2. Solve for l_∞

Since u, v is obtained, from [8], we have,

$$l_\infty{}^2 = C_1 + \mu C_2 = l^2 + u l_1 l_2 + v l_3 l_4 , \quad (8)$$

where l_∞ is the vanishing line of the supporting plane of the two concentric circles. The solution of l_∞ is the eigenvector of $C_1 + \mu C_2$ corresponding to the maximum eigenvalue.

3.3. Homological relation between the two ICC

From [9], [4], [6], we know that there exists a homological relationship between the two ICC. If the homology between the two ellipses C_1, C_2 is denoted by G, we have,

$$C_1 = G^T C_2 G . \quad (9)$$

Each homology has a fixed line and a fixed point. In the case of the two ICC, the fixed line is the vanishing line l_∞, and the fixed point is the image of the center of the circles denoted as O' in the Figure. 1. Therefore, the homology G can be represented by its fixed line l_∞, fixed point O' and a cross- ratio ρ (which can be obtained from the ratio of the radii of the two concentric circles) [9] [4] [6]:

$$G = I_{3\times3} + \rho \frac{O' l_\infty^T}{O'^T l_\infty} . \quad (10)$$

3.4. Solve for w

The vanishing line l_∞ and the image of the center O' have a pole-polar relationship with respect to the image of the outer circle C_1 [9] [4], then we have:

$$O' = C_1^{-1} l_\infty , \quad (11)$$

or

$$l_\infty = C_1 O' . \quad (12)$$

Since P_{12}, P_{34} and O' are collinear, we have:

$$O' = P_{12} + \lambda P_{34} , \quad (13)$$

where λ is an unknown constant. From (3) and (13),

$$\begin{aligned} C_1 O' &= \left(w l^2 + u l_1 l_2\right)\left(P_{12} + \lambda P_{34}\right) \\ &= w l^2 P_{12} + w l^2 \lambda P_{34} + u l_1 l_2 P_{12} + u l_1 l_2 \lambda P_{34} \end{aligned} . \quad (14)$$

Since , P_{12} lies on l_1 and l_2, then

$$l_1 l_2 P_{12} = 0 . \quad (15)$$

Therefore,

$$C_1 O' = w l^2 P_{12} + w \lambda l^2 P_{34} + \lambda u l_1 l_2 P_{34} . \quad (16)$$

From (12) and (16), we have,

$$l_\infty = w l^2 P_{12} + w \lambda l^2 P_{34} + \lambda u l_1 l_2 P_{34} . \quad (17)$$

Since l_∞, l, P_{12}, P_{34}, and u are all known, there are only two variables, λ and w, are unknown. Then from (17), we can obtain two equations but in fact only one independent equation on λ and w. The equation can be denoted as:

$$\alpha \lambda w + \beta \lambda + \gamma w = 0 , \quad (18)$$

where α, β, γ are some coefficients determined by l_∞, l, P_{12}, P_{34}, and u. Therefore, we have:

$$\lambda = \frac{-\gamma w}{\alpha w + \beta} . \quad (19)$$

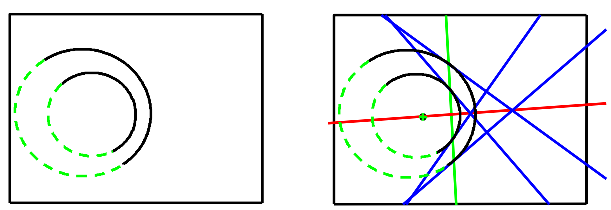
Substitute (19) into (13), then (13) into (10), then (10) into (9), and with (3) and (4), we can obtain 6 equations on w. Therefore w can be solved from these equations using some least squares method.

After u, v, and w are obtained, from (5) and (6) we can determine ζ and ξ. Therefore from (1) and (2), we can determine the equations for the two ICC.

4. Experiments

4.1. Simulated data

We use Figure. 2 as a simulated example to show the detection results for the proposed method. An image containing projected concentric circles is shown in Figure. 2a, where the image size is 800×600. Gaussian noise with zero-mean and standard deviation 1 is added to these visible image points. Then, we use the proposed method to recover the images of the two concentric circles. The detection results are shown in Figure. 2b.

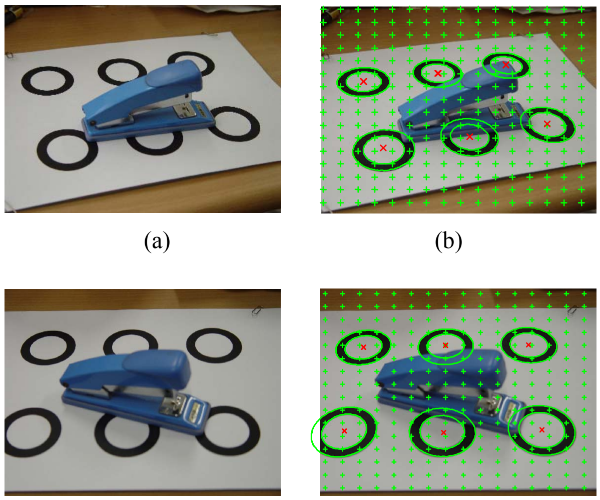


(a) (b)

Figure. 2. (a) A simulated image containing two projected concentric circles with occlusions. (b) The detection of the image of the common center is shown in 'x'.

4.2. Real data

Some real images containing one or more images of pairs of projected concentric circles with the effects of occlusion are shown in Figure. 3a, and 3c. These images are captured using a Sony digital camera DSC-F717, and the image size is 1024×768. There are many edge detection techniques available. The one proposed in [7] can be used to obtain edge segments without branches. Among the resulting edge segments, we discard short ones and register the remaining ones in an edge segment list. After finding the edges in the image, the method proposed in this paper is employed, and the detected ellipses and the images of the common centers are superimposed onto the original images as shown in Figure. 3b, and 3d.



(a) (b) (c) (d)

Figure. 3. The experiments with a pattern of six pairs of concentric circles. The seed points are shown in '+', and the final recovered imaged circle centers in 'x'.

5. Conclusions

This paper discovered that only four intersection edge points on a secant line of the two ICC are sufficient to determine the whole parameters of the ICC, when the ratio of the radii of the two concentric circles is given. The main contribution of this paper may be that we use the minimum number, four edge points on a secant line, not eight on two secant lines as used in previous methods, to recover the whole parameters of the two ICC, which makes the proposed method more suitable to deal with serious occlusions. To investigate the case that the four nonlinear edge points on the two ICC are used to perform detection is our ongoing work.

Acknowledgements

This work was supported in part by the NKBRPC 973 Grant No. 2006CB303100, the NNSFC Grant No. 60605010, and the NHTRDP 863 Grant No. 2006AA01Z302.

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